

Semantics lab class (Course 2)

Attitude predicates and intensionality

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Our agenda today

- Something new:
Attitude predicates and intensionality, Accessibility relations
Reflexivity and veridicality
- Some exercise to help you with assignment 7

Extensional semantics

This is our **extensional** system up to this point:

- We have identified the denotation of sentences with their actual **truth-values** in relation to **the real world**.
- The **extension** of a complex expression can be computed from the **extensions** of its parts.

Recall: One can only decide the truth or falsity of the sentence in a given situation.

(1) Snowball is on the sofa.

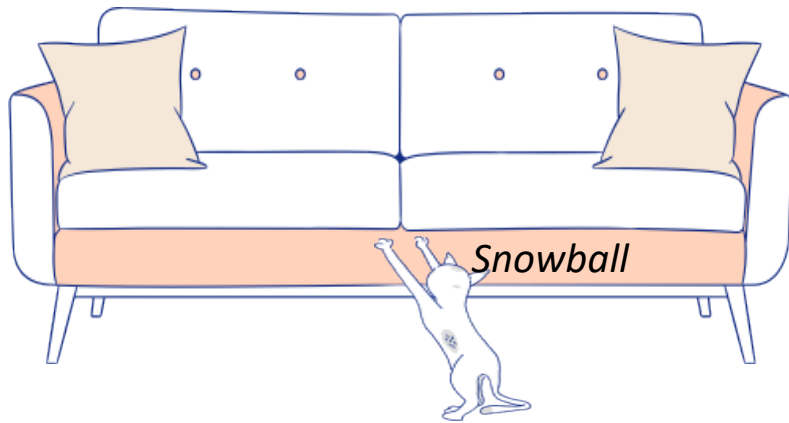
To determine whether (1) is true or false, you have to inspect the situation in the real world and know

a) who Snowball is

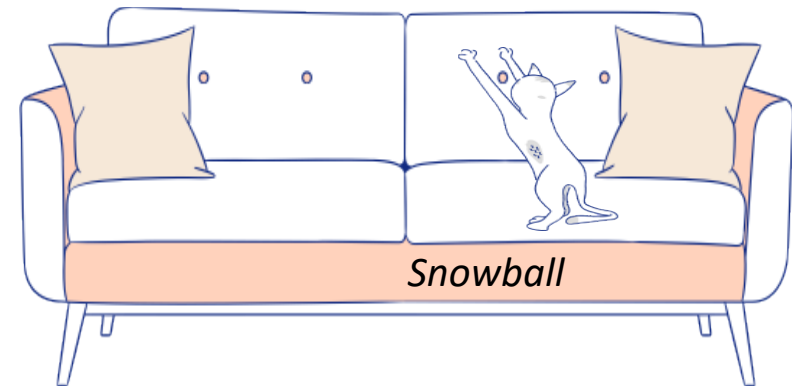
b) whether Snowball is on the sofa.

If in the real world:

Situation S_1 , (1) is false.



In Situation S_2 , (1) is true.



Problems with extensional semantics

However, the above assumptions are problematic for examples of the following kind:

(2) a. Burt believes that_{s1} [it is raining in New York.]

b. Burt believes that_{s2} [it is snowing in Berlin.]

Where the extensional semantics breaks down: Problem 1

- (2) a. Burt **believes** that s_1 [it is raining in New York.]
b. Burt **believes** that s_2 [it is snowing in Berlin.]

Let the situation S be that it is raining in New York and it is snowing in Berlin .

Then $[[_{s_1} \text{ it is raining in New York}]] = [[_{s_2} \text{ it is snowing in Berlin}]] = 1$.

Since $[[\text{believes}]]$ and $[[\text{Burt}]]$ will be assigned the same denotation, then we predict that :

$$[[\text{Burt believes that } S_1]] = [[\text{Burt believes that } S_2]]$$

The equation doesn't hold. Burt can believe it is raining in New York, but at the same time doesn't believe it is snowing in Berlin.

Our current system makes the wrong prediction.

Attitude verbs

In human languages, we often report the mental state or a communicative act of some individual: What someone believes, wants, hopes, says, etc.

(3) a. Burt **believes** that it is raining in New York.

b. Joan **wants** to go to the movies.

c. Peggy **hopes** that she will get a raise.

mental state

(4) a. Pete **said** that the bagel was tasty.

b. Don **promised** that he would be home for dinner.

c. Roger **claimed** that his car had broken down.

communicative acts

These two classes of verbs are called **attitude verbs**.

Where the extensional semantics breaks down:

Problem 2

Standardly we evaluate a sentence in the [here and now](#).

[[Chomksy smokes]]= 1 iff Chomksy smokes ([here and now](#)).

But human languages are not restricted to discourse about the actual here and now. The shifting of the evaluation is called **displacement**.

(5) a. [In Hamburg](#), it is raining right now.

Spatial Displacement

b. [A few days ago](#), it rained.

Temporal Displacement

c. [If the low pressure system had not moved away](#), it might have been raining now.

Modal Displacement

Intensionality as evaluation shifting

The sentences with displacement create **non-extensional contexts**, which cannot be captured by our current semantics.

We need to move to a semantics that is *intensional*. This means:

It has to contain operators that “displace” the evaluation of a sentence from the actual here and now to other points of reference (spatially, temporally, and modally)

Intensionality: Evaluation relative to worlds

Intension as a **function** relativizes meaning:

Input: A package of the various factors which can determine the extension
(spatial, temporal, mental state
and modal...)



Output: An appropriate extension

Within the scope of this lecture, we will focus on only one kind of the input package:
Possible worlds.

What is a world

We use the technical notion of “world” in a way that is as **inclusive** as possible.

When we choose a world w as evaluation world for a sentence S , it means **the whole record of w** is relevant for evaluation of S :

- everything there ever was in w ,
- everything there presently is in w , and
- everything there ever will be in w

Individuals and **entities** more generally are parts of worlds.

Possible worlds Semantics

w is assumed to be **the actual world**, often notated as w_0 .
However, things might have been different in countless ways, and **each different way of putting everything together is a possible world**.

Examples:

- In w_0 , Chomsky is a linguist who smokes.
In w_2 , Chomsky is a singer who never smokes.
- In w_0 , one third of DB trains in Germany arrived late in 2022.
In w_1 , No DB trains in Germany arrived late in 2022.

For simplicity, we will stay away from the metaphysical debate.

Defining extension and intension

Extension:

In our old extensional semantics, the notation “ $[[\alpha]]$ ” was read as “the denotation of α ”, or “the extension of α (in w_0)”.

In our new system, **the extension is world dependent.**

We read $[[\alpha]]^w$ as “the extension of α with respect to a given world of evaluation w ”.

Examples:

$[[\text{Clemens smokes}]]^w = 1$ iff Clemens smokes in w .

$[[\text{singer}]]^w = \lambda x_e. x$ is a singer in w

Defining extension and intension

Intension

The intension is a function from worlds to extensions.

The intension is independent of any given world,

$$\lambda w. [[\alpha]]^w$$

(abbreviated as $[[\alpha]]_\phi$)

Examples:

The intension of “*Clemens smokes*”: $\lambda w. \text{Clemens smokes in } w$

The intension of “*singer*”: $\lambda w. \lambda x_e. x \text{ is a singer in } w$

Intensional Domains

In our old system, there are two basic types: **e**, **t**

$D_e = D$, the set of all possible individuals

$D_t = \{0,1\}$, the set of truth-values

We expand the set of semantic types by adding a new basic type, **the type s**.

$D_s = W$, the set of all possible worlds

We now have three basic types: **e**, **t**, and **s**.

Extension vs. intension

Expression type	Lexical entries	Extension	Intension
Proper names	$[[\text{Tony}]]^w = \text{Tony}$	individual: e	individual concept: $\langle s, e \rangle$
Predicates	$[[\text{smoke}]]^w = \lambda x \in D_e . x \text{ smokes in } w$ $[[\text{criminal}]]^w = x \in D_e . x \text{ is a criminal in } w$	Predicates: $\langle e, t \rangle$	property: $\langle s, \langle e, t \rangle \rangle$
Sentences	$[[\text{Tony smokes}]] = 1 \text{ iff Tony smokes in } w$	truth-value: t	proposition: $\langle s, t \rangle$

World (in)dependence

Expression type	Lexical entries	World (in)dependency of extension
Proper names	$[[\text{Tony}]]^w = \text{Tony}$	World independent. Rigid designators: They denote the same individual in every possible world
Predicates	$[[\text{smoke}]]^w = \lambda x \in \text{De} . x \text{ smokes in } w$ $[[\text{criminal}]]^w = \lambda x \in \text{De} . x \text{ is a criminal in } w$	<i>World dependent.</i> The denotation of predicates differs from world to world.
Sentences	$[[\text{Tony smokes}]] = 1 \text{ iff Tony smokes in } w$	<i>World dependent.</i>
Logical operators	$[[\text{not}]]$ $[[\text{and}]]$ $[[\text{or}]]$	<i>World independent.</i>

Semantic rules with evaluation parameter

Two parameters: **assignment functions** and **evaluation worlds**.

Accordingly, we need to relativize the interpretation function $[[\]]$ to both parameters:

$$[[\alpha]]^{w,a}$$

Recall **AID**:

When α is assignment-independent, $[[\alpha]]^{w,a} = [[\alpha]]^{w,\emptyset} = [[\alpha]]^w$

Semantic rules with evaluation parameter

- TN1 If α is a terminal node, then for any possible world w α is in the domain of $[]^w$ if $[\alpha]^w$ is specified in the lexicon.
- TN2 If α is an index i , then for any possible world w and any assignment a , $[i]^{w,a} = a(i)$.
- NN If α is a non-branching node, and β is α 's daughter, then for any possible world w and any assignment a , α is in the domain of $[]^{w,a}$ if β is in the domain of $[]^{w,a}$. Then $[\alpha]^{w,a} = [\beta]^{w,a}$.
- PM If α is a branching node and $\{\beta, \gamma\}$ is the set of α 's daughters, then for any possible world w and any assignment a , α is in the domain of $[]^{w,a}$ if β and γ are in the domain of $[]^{w,a}$ and $[\beta]^{w,a}$ and $[\gamma]^{w,a}$ are both in $D_{\langle e,t \rangle}$. Then $[\alpha]^{w,a} = \lambda x \in D_e . [\beta]^{w,a}(x) = [\gamma]^{w,a}(x) = 1$.
- FA If α is a branching node and $\{\beta, \gamma\}$ is the set of α 's daughters, then for any possible world w and any assignment a , α is in the domain of $[]^{w,a}$ if β and γ are in the domain of $[]^{w,a}$ and $[\gamma]^{w,a}$ is in the domain of $[\beta]^{w,a}$. Then $[\alpha]^{w,a} = [\beta]^{w,a}([\gamma]^{w,a})$.

To understand “believe”

Now let's go back to attitude verbs. Recall the problem we encountered, repeated below:

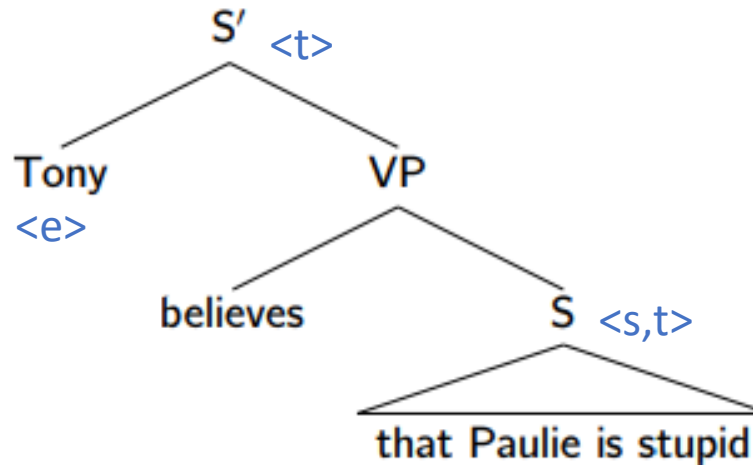
- (1) a. Burt **believes** that _{s₁} [it is raining in New York.]
b. Burt **believes** that _{s₂} [it is snowing in Berlin.]

Our old semantics predicts that (1a) and (1b) are equivalent.

Such problem tells us that the meaning of “*believe*” cannot be defined purely based on the extension of its embedded clause, i.e. not of type $\langle t, \langle e, t \rangle \rangle$.

Believe does not express a relation between an individual (Burt) and a truth value (the extension of sentences).

To understand “believe”



$[[\text{believes}]]^w$ is a function that maps:

propositions of type $\langle s, t \rangle \rightarrow$

A function VP from an individual (the belief holder) $\langle e \rangle$ to a truth-value $\langle t \rangle$.

$[[\text{believes}]]^w$ should be of type $\langle \langle s, t \rangle, \langle e, t \rangle \rangle$

To understand “believe”

What are beliefs?

Intuitively, beliefs represent ways that things are, according to the belief holder.

But our beliefs simply leave too many questions unsettled.

To understand “believe”

For example, assume Tony doesn't know what kind of person Paulie is. The best he can do is have *a set of candidate worlds according to his beliefs* W_T^B for the actual world w_0 :

w_1 : Paulie is smart in w_1 .
 w_2 : Paulie is stupid in w_2 .
 w_3 : Paulie is not stupid in w_3 .
.....

W_T^B

If Tony believes that Paulie is stupid, this means w_1 and w_3 must be excluded from Tony's set of candidate worlds W_T^B . We say, w_1 and w_3 are not **compatible** with what Tony believes in the actual world w .

To understand “believe”

~~w_1 : Paulie is smart in w_1 .~~ —————

W_T^B

w_2 : Paulie is stupid in w_2 .

~~w_3 : Paulie is not stupid in w_3 .~~

.....

If Tony believes that Paulie is stupid, this means w_1 and w_3 must be excluded from Tony's set of candidate worlds W_T^B .

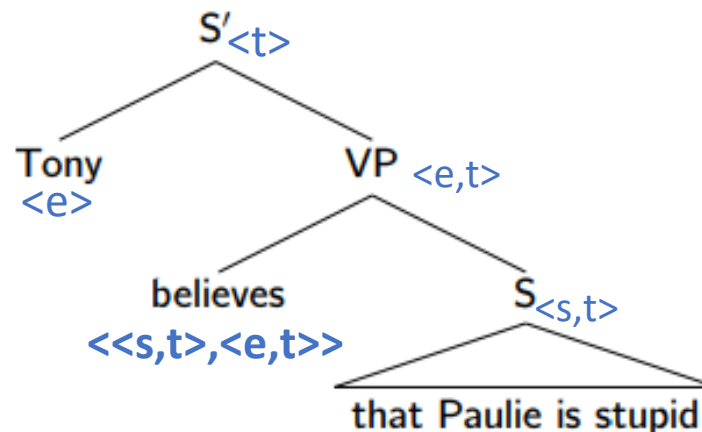
We say, w_1 and w_3 are **not compatible** with what Tony believes in the actual world w_0 .

Lexical entry of *believe*

The truth conditions of a belief report can thus be stated as in (5):

(5) $[[\text{Tony believes } S]]^w = 1$ iff every possible world w' of Tony's set of candidates W_T^B is compatible with what x believes in w .

$$[[\text{believe}]^w = \lambda p \in D_{\langle s, t \rangle}. [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow p(w') = 1]]$$



A new rule: Intensional functional application (IFA)

Now our semantics requires *believe* to take an intension of type $\langle s, t \rangle$ as an argument.

But our current rule FA only takes extension as argument.

FA If α is a branching node and $\{\beta, \gamma\}$ is the set of α 's daughters, then for any possible world w and any assignment a , α is in the domain of $\llbracket \cdot \rrbracket^{w,a}$ if β and γ are in the domain of $\llbracket \cdot \rrbracket^{w,a}$ and $\llbracket \gamma \rrbracket^{w,a}$ is in the domain of $\llbracket \beta \rrbracket^{w,a}$.
Then $\llbracket \alpha \rrbracket^{w,a} = \llbracket \beta \rrbracket^{w,a}(\llbracket \gamma \rrbracket^{w,a})$.

A new rule: Intensional functional application (IFA)

We must introduce a new **intensional** FA (IFA).

IFA If α is a branching node and $\{\beta, \gamma\}$ is the set of α 's daughters, then for any possible world w and any assignment a , if $\llbracket \beta \rrbracket^{w,a}$ is a function whose domain includes $\llbracket \gamma \rrbracket_{\phi}^a$, $\llbracket \alpha \rrbracket^{w,a} = \llbracket \beta \rrbracket^{w,a}(\llbracket \gamma \rrbracket_{\phi}^a)$.

Computing the truth-conditions

$\llbracket \text{that Paulie is stupid} \rrbracket^w = 1$ iff Paulie is stupid in w

$\llbracket \text{believes that Paulie is stupid} \rrbracket^w$

$= \llbracket \text{believes} \rrbracket^w(\llbracket \text{that Paulie is stupid} \rrbracket_\phi)$ (IFA)

$= [\lambda p \in D_{\langle s, t \rangle}. [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow p(w') = 1]]](\llbracket \text{that Paulie is stupid} \rrbracket_\phi)$

$= [\lambda p \in D_{\langle s, t \rangle}. [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow p(w') = 1]]](\llbracket \lambda w''. \llbracket \text{that Paulie is stupid} \rrbracket^{w''} \rrbracket)$

$= [\lambda p \in D_{\langle s, t \rangle}. [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow p(w') = 1]]](\llbracket \lambda w''. \text{Paulie is stupid in } w'' \rrbracket)$

$= [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow \llbracket \lambda w''. \text{Paulie is stupid in } w'' \rrbracket(w') = 1]]$

$= [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow \text{Paulie is stupid in } w']]$

$\llbracket \text{Tony believes that Paulie is stupid} \rrbracket^w$

$= 1$ iff $\forall w' [w' \text{ is compatible with what } T \text{ believes in } w \rightarrow \text{Paulie is stupid in } w']$

Exercise 1: Propositional attitude predicates

Doubt is definable via *believe*.

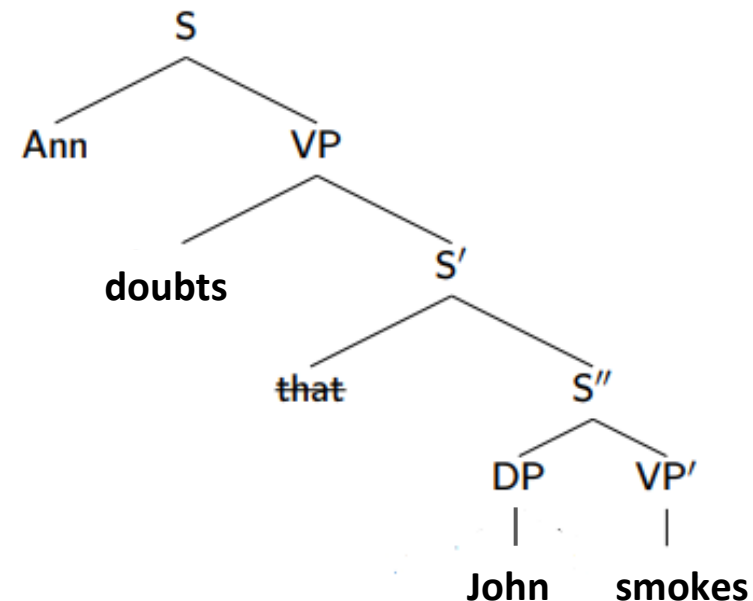
$$[[\text{doubt}]]^w = \lambda p_{\langle s, t \rangle} . [\lambda x_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow p(w') = 0]]$$

Compute the truth-conditions of (6).

(6) Ann doubts that John won.

Remember both **evaluation parameters**:

Assignments and worlds.



Exercise 1: Solutions

$$\begin{aligned}
 [[S']]^{w,g} &= [[S']]^w && \text{(AID)} \\
 &= [[VP']]^w ([[DP]]^w) && \text{(FA)} \\
 &= [[smokes]]^w ([[John]]^w) && \text{(2xNN)} \\
 &= [\lambda x \in De . x \text{ smokes in } w] (\text{John}) && \text{(2xTN1)} \\
 &= 1 \text{ iff John smokes in } w
 \end{aligned}$$

$$\begin{aligned}
 [[VP]]^{w,g} &= [[doubts]]^{w,g} (\text{[[S']]^{g}_{\phi}}) && \text{(IFA)} \\
 &= [[doubts]]^w ([[\lambda w'. [[S']]^{w'}]]) && \text{(AID)} \\
 &= [\lambda p \in D_{\langle s,t \rangle} . [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow \\
 &\quad p(w') = 0]]] ([\lambda w''. [[S']]^{w''}]) && \text{(TN1)} \\
 &= [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow [\lambda w''. [[S']]^{w''}] \\
 &\quad (w')=0]] \\
 &= [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow [[S']]^{w'} = 0]]
 \end{aligned}$$

Exercise 1: Solutions

$$\begin{aligned} [[S]]^{w,g} &= [[VP]]^{w,g}([Ann]^{w,g}) && (FA) \\ &= [[VP]]^w([Ann]^w) && (AID \times 2) \\ &= [\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ believes in } w \rightarrow \\ &[[S']]^{w'} = 0]] (Ann) && ([[VP]]^w, TN1) \\ &= 1 \text{ iff } \forall w' [w' \text{ is compatible with what Ann believes in } w \rightarrow \\ &[[S']]^{w'} = 0] \\ &= 1 \text{ iff } w' \text{ is compatible with what Ann believes in } w \rightarrow \text{John} \\ &\text{smokes in } w' = 0 \end{aligned}$$

Accessibility relation

Just like *believe*, other attitude verbs (*hope*, *know*, *expect*) also quantify over a set of possible worlds as candidates for w .

$$[[\text{know}]]^w = \lambda p_{st} . [\lambda x_e . \forall w' [w' \text{ is compatible with what } x \text{ knows in } w \rightarrow p(w') = 1]]$$

$$[[\text{expect}]]^w = \lambda p_{\langle s, t \rangle} . [\lambda x_e . \forall w' [w' \text{ is compatible with what } x \text{ expects in } w \rightarrow p(w') = 1]]$$

$$[[\text{hope}]]^w = \lambda p_{\langle s, t \rangle} . [\lambda x_e . \forall w' [w' \text{ is compatible with what } x \text{ hopes in } w \rightarrow p(w') = 1]]$$

In what ways are these attitude verbs different?

Accessibility relation

We say the belief worlds are the worlds **accessible** given x 's beliefs in w .

$$\llbracket \text{believe} \rrbracket^w = \lambda p_{\langle s, t \rangle} . [\lambda x_e . \forall w' [w \mathcal{R}_x^B w' \rightarrow p(w') = 1]]$$

The sole difference between various attitudes is in the **accessibility relation** that determines the set of worlds they quantify over.

$$\llbracket \text{hope} \rrbracket^w = \lambda p_{\langle s, t \rangle} . [\lambda x_e . \forall w' [w \mathcal{R}_x^H w' \rightarrow p(w') = 1]]$$

$$\llbracket \text{expect} \rrbracket^w = \lambda p_{\langle s, t \rangle} . [\lambda x_e . \forall w' [w \mathcal{R}_x^E w' \rightarrow p(w') = 1]]$$

$$\llbracket \text{know} \rrbracket^w = \lambda p_{\langle s, t \rangle} . [\lambda x_e . \forall w' [w \mathcal{R}_x^K w' \rightarrow p(w') = 1]]$$

Reflexivity and veridicality

The concept of **knowledge** crucially contains the concept of **truth**: what we know must be true.

(7) Jane knows (for a fact) she passed the exam. **?** But it is not the case she passed the exam.

If in w we know that something is the case then it must be the case in w . So, w must be compatible with all we know in w . We say \mathcal{R}_x^K is **reflexive**.

However, we can have false beliefs. \mathcal{R}_x^B is **not reflexive**.

(8) Jane believes she passed the exam. But in fact it is not the case she passed the exam.

Reflexivity and veridicality

Reflexive accessibility relation licenses the **inference** that the embedded clause is **true**. This is called **veridicality**.

- (9) a. I **know** p . Inference: p must be true.
(**reflexive, veridical**)
- b. I **believe** p . No Inference that p must be true.
(**non-reflexive, non-veridical**)
- c. I **hope** p . No Inference that p must be true.
(**non-reflexive, non-veridical**)
- d. I **doubt** p . No Inference that p must be true.
(**non-reflexive, non-veridical**)

Exercise 2: Accessibility relation

1) What kind of possible accessibility relation do the following attitude predicates involve? What inferences do these sentences have?

2) Try to come up with a lexical entry for *glad*.

- (10)
- a. John **assumed** that it is raining outside.
 - b. That John won the game **shocked** me.
 - c. Mary was **glad** that John called her.
 - d. John **regretted** that he left the party.
 - e. Mary **found out** that John left the party.

Exercise 2: Solutions

(10) a. John **assumed** that it is not raining outside. ✂️ It is raining outside.

assume's accessibility relation is non-reflexive, non-veridical

b. That John won the game **shocked** me. ~~~~~ John won the game.

shock's accessibility relation is reflexive and veridical.

c. Mary was **glad** that John called her. ~~~~~ John called Mary.

glad's accessibility relation is reflexive and veridical.

d. John **regretted** that he left the party. ~~~~~ He left the party.

regret's accessibility relation is reflexive and veridical.

e. Mary **found out** that John left the party. ~~~~~ John left the party.

find out's accessibility relation is reflexive and veridical.

Exercise 2: Solutions

Emotive factives like “glad” “happy” “surprised” have their accessibility relation presumably based on **mood**.

There is a fact that John visited Mary (hence that John visited Mary is true), and this fact made Mary happy.

Two possible lexical entries:

$[[\text{glad}]]^{w,a} = \lambda p \in D_{\langle s,t \rangle}$ and $p(w) = 1$. $[\lambda x \in D_e . \forall w' [w' \text{ is compatible with what } x \text{ would feel glad about in } w \rightarrow p(w') = 1]]$

$[[\text{glad}]]^{w,a} = \lambda p \in D_{\langle s,t \rangle}$ and $p(w) = 1$. $[\lambda x \in D_e . \forall w' [w \mathcal{R}_x^G w' \rightarrow p(w') = 1]]$