Semantics lab class (Course 2)

Lecture 3, assignment 3

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Session 2

November 15, 2023

Our agenda today

- A recap of what we left unexplained last time:

Sets, functions, rules for compositional interpretation

- Review assignment 2

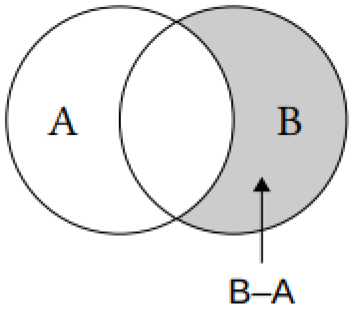
- Also something new:

transitive verbs, characteristic functions, schonfinkelization

Basic concepts in set theory: Check list

Set relations:

- Equivalence A = B, iff A and B have exactly the same elements.
- Subset A ⊆B, iff all members of A are members of B.
- Proper Subset A ⊂ B, iff A is a subset of B but not equivalent to B
- Intersection A ∩ B, is the set C with exactly those elements which are shared by A and B.
- Overlap and disjointness A and B overlap iff A ∩ B ≠ Ø A and B are disjoint if A ∩ B = Ø
- Union A∪B, the C with all the members of A and B and nothing else.
- Complement The complement of A relative to B, B–A



Exercise 1: Set relations

(1) True or false?

a. $\{a\} = \{b\}$

b. $\{x: x=a\} = \{a\}$

c. $\{x: x \text{ is green}\}=\{y: y \text{ is green}\}$

d. $\{x: x \text{ likes } a\} = \{y: y \text{ likes } b\}$

e. $\{x : x \in \{y : y \in B\}\} = B$

(2) True or false?

a. $\{a,b,c\} \subseteq \{a,b,c\}$

b. $A \subset A$

c. If $A \subseteq B$ and $B \subseteq A$, then $A=B$

d. For every set S, $\emptyset \subseteq S$

e. i. $\{a,b\} \in \{\{a,b\}, \{c,d\}\}$ ii. $\{a,b\} \subseteq \{\{a,b\}, \{c,d\}\}$ iii. $\{a,b\} \in \{a,b,c,d\}$

Solutions for (1)

(1) True or false?

a. {a} = {b} — True when a=b

b. {x: x=a} = {a} — True

c. {x: x is green}={y: y is green} — True

d. {x: x likes a} = {y: y likes b} — True when a=b

e. {x : x ∈ {y : y ∈ B}} = B — True

Solutions for (2)

(2) a. True. Every set is a subset of itself.

b. False. Every set can't be a proper subset of itself.

c. True. $A \subseteq B$ means, all members of A are members of B.

$B \subseteq A$ means, all members of B are members of A.

Hence, $A = B$.

d. True.

e. True, false, false

Tips for subset vs. set membership

<u>When in doubt, go back to the basic definitions of set and set realtions.</u>

<u>Subset</u> A ⊆B, iff all members of A are members of B.

Trick: When you see "⊆", list all members of set A and B and compare them.

<u>Members/elements: </u> x ∈ B : "x is a member (or element) of set B".

Trick: When you see "∈", list all the members of B and check if x is one of them.

Solutions for (2e)

i. $\{a,b\} \in \{\{a,b\}, \{c,d\}\}$

True. $\{\{a,b\}, \{c,d\}\}$ has two elements $\{a,b\}$ and $\{c,d\}$.

ii. $\{a,b\} \subseteq \{\{a,b\}, \{c,d\}\}$

False. $\{a,b\}$ has two elements a and b. $\{\{a,b\}, \{c,d\}\}$ has two elements $\{a,b\}$ and $\{c,d\}$.

iii. $\{a,b\} \in \{a,b,c,d\}$

False. $\{a,b,c,d\}$ has four elements a,b,c and d. $\{a,b\}$ is not one of them.

Assignment 2

Any questions?

When a=1, b=1, a=b.   When A={1}, B={1}, A=B

But *x,y* are variables, not names. ~~x=y~~

Variables are called variables because they vary, i.e. they can have a variety of values.

A:= {x: x is green}={y: y is green}= {z: z is green}= {$: $ is green}={abc: abc is green}

- x, y, z are variables ranging over objects
- a, b, c are names of objects
- X, Y, Z are variables ranging over sets
- A, B, C are names of sets

Assignment 2

Exercise 1 (1b)

{x: {y : y likes x} = {Mary} } = {x : {y : x likes y} = {Mary}}

Read the inner layer first.

Why functions?

We looked at sets of individuals.

What about sets of couples (or triples, quadruples,etc.) of individuals?

Example: The set of all married couples who live in New York.

{Will & Dorothy Bradford, John & Mary Brewster, May & Rose Standish, Edward & Bill Winslow, …}

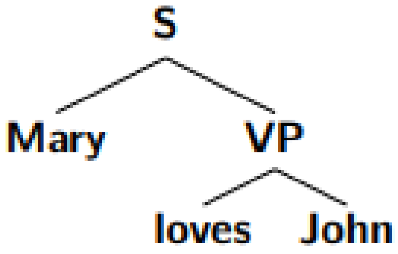
In this example, the set is defined as a set of pairs, but the order doesn't matter.

Ordered pairs

In certain cases, specifying the order is important.

Consider sentence (3) with the transitive verb “love”:

(3)



Note: <Mary, John> ≠ <John, Mary>

Relations

A set of ordered pairs is called a **relation**. Relations are defined by a condition to the right of ':'

(4) *Situation: Mary loves John and Jane. John loves Jane. Jane loves Sue.*

R:= {<x, y>: x loves y}

<Mary, John> ∈ R

<Mary, Jane> ∈ R

<John, Jane> ∈ R

<Jane, Sue> ∈ R <Sue, Jane> ∉ R

R:= {<Mary, John>, <Mary, Jane>, <John, Jane>, <Jane, Sue>}

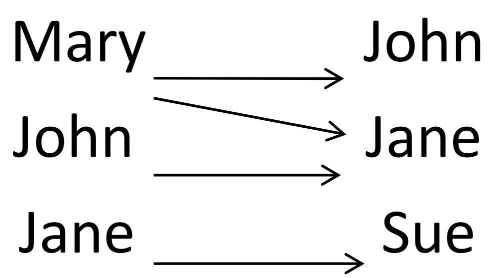
The **domain** of R is the set {Mary, John, Jane}.

The **range** of R is the set {John, Jane, Sue} .

Relations define mappings

A set of ordered pairs can be seen as **mapping,** a correspondence from the domain onto the range.

(4) R:= {<Mary, John>, <Mary, Jane>, <John, Jane>, <Jane, Sue>}

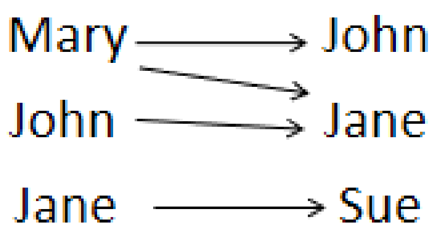


Functions and its uniqueness

First of all, a **Function is a relation.**

But, unlike relations, for a function, each element of the domain is mapped to a **single, unique value** in the range.

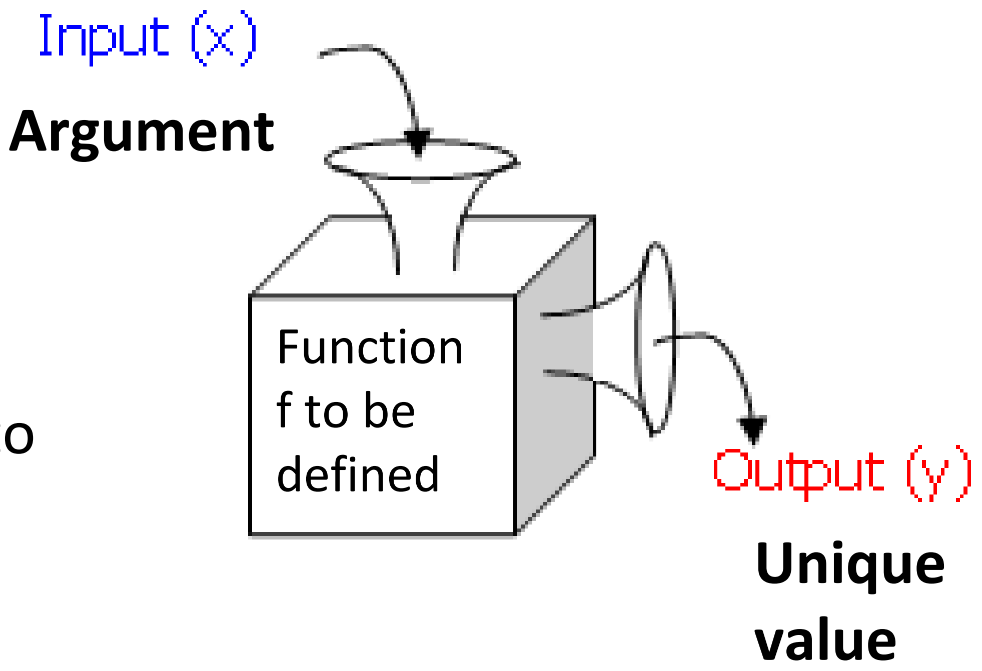
(4) R:= {<Mary, John>, <Mary, Jane>, <John, Jane>, <Jane, Sue>}



(16) is not a function; Two distinct ordered pairs have the same first element <Mary, John>, <Mary, Jane>.

A tip for understanding functions

If we set our machine to $y=f(x)=x+1$ ...



$f(x)$ = 'the value of f for the argument x'.

Defining functions

- via listing:

$$F := \{\langle a, b\rangle, \langle b, c\rangle, \langle c, a\rangle\}$$

$$=$$

$$F := \begin{bmatrix} a \to b \\ b \to c \\ c \to a \end{bmatrix}$$

- with conditions (3 types):

$F := \{\langle x, y\rangle : x$ is a French president and y is x's year of birth$\}$

Let F be that function f such that $f : \{x : x$ is a French president$\} \to \{x : x$ is a year$\}$, and for every $x \in \{x : x$ is a French president$\}$, $f(x) =$ the year in which x was born.

$$F := \begin{array}{l} f : \{x : x \text{ is a French president}\} \to \{x : x \text{ is a year}\} \\ \text{For every } x \in \{x : x \text{ is a French president}\}, f(x) = \text{the year } x \text{ was born.} \end{array}$$

Exercise 2: Functions/relations

(5) Function or not?

a. R_1:= {<x,y>: y=2x+1} (where x and y are integers)

b. R_2:={<x,y>: x is a human being and y is x's birth mother}

c. R_3:= {<x,y>: x is a human being and y is x's son}

d. R_4:= {<y,x>: x is a human being and y is x's son}

e. Assume we have two sets. A = {a,b,c} and B = {1,2,3,4}. The following relations from A to B:

R_5:= {<a,1>, <b,2>, <c,3>}

R_6:= {<a,3>,<b,4>,<c,1>, <a,2>}

R_7:= {<a,1>,<b,2>}

Solutions: Exercise 2

(5) a. R_1 is a function.

b. R_2 is a function. Human beings can only one birth mother.

c. R_3 is a not function. Human beings can more than one sons.

d. R_3 is a not function. One son can be mapped to more than one parent.

e. R_5 is a function.

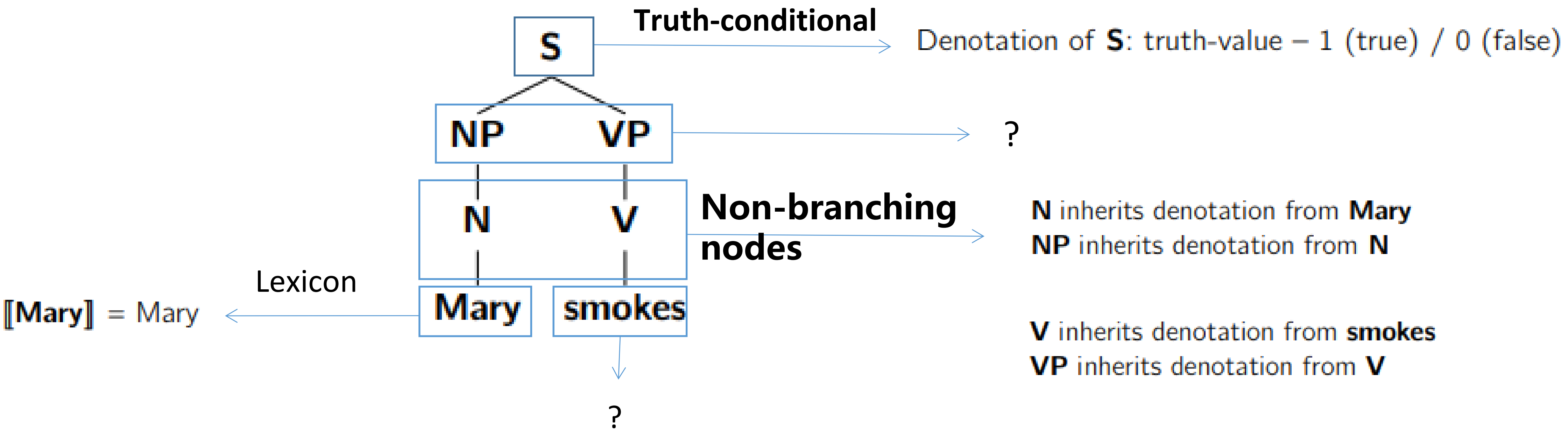
R_6 is not a function. The argument *a* can be mapped to more than one value *2* and *3*.

R_7 is not a function. All arguments within the domain D should be mapped to a value.

How does compositional interpretation work?

<u>Truth-conditional semantics:</u> Knowing the meaning of an expression consists in knowing the conditions under which it is true.

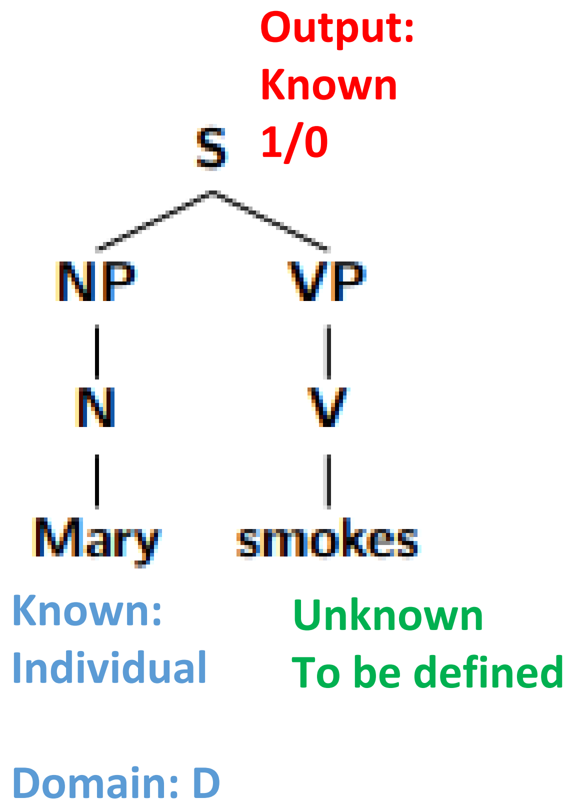
(6) **Mary smokes.**

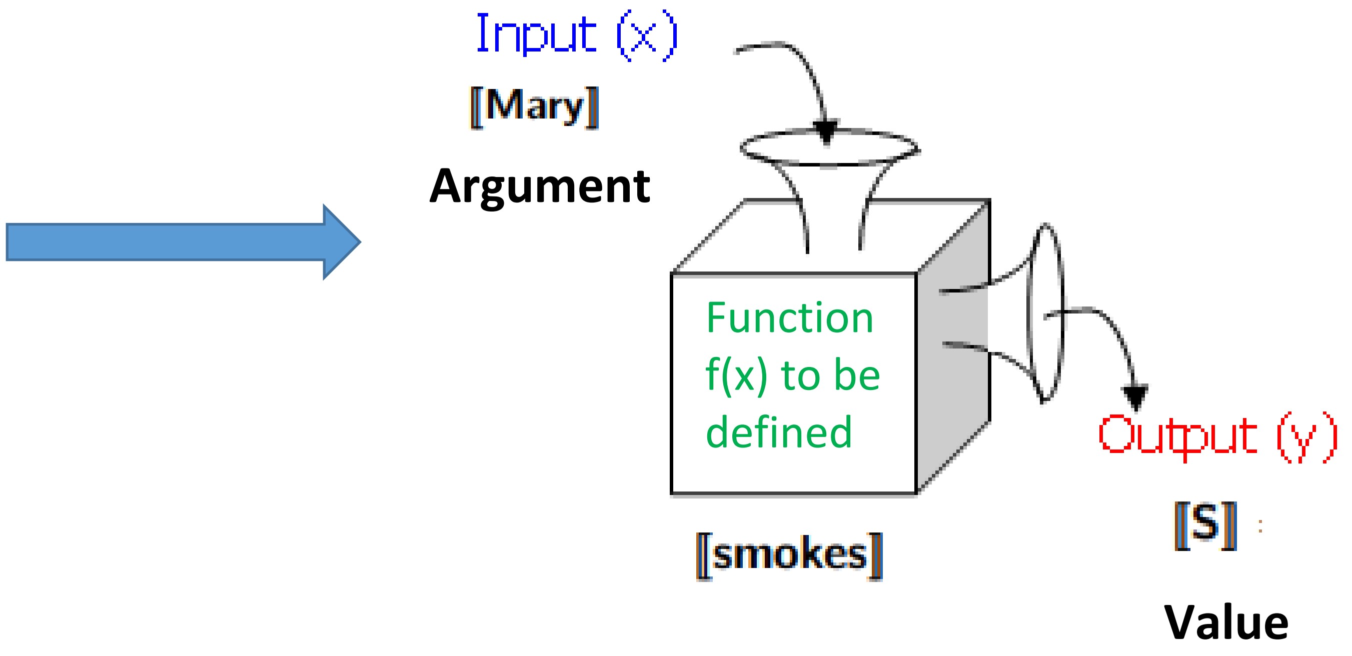


What remains unclear

[[S]] = [[VP]]([[NP]]) How do we combine two braches?

[[VP]]= ? How do we understand verbs?





Function Application (FA) as a model

Our solution: Using **function** as a tool to interpret any syntactic structure with two branches.

If α has the form $[_{S}\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

One branch is interpreted as a **function**, and the other branch is interpreted as a possible **argument** of the function.

$$[\![S]\!] = [\![VP]\!]([\![NP]\!]) \qquad (S1)$$

$$= [\![smokes]\!]([\![Mary]\!]) \qquad ([\![VP]\!], [\![NP]\!])$$

Function | Argument

$$= \left[\begin{array}{l} f : D \rightarrow \{0,1\} \\ \text{For all } x \in D, f(x) = 1 \text{ iff } x \text{ smokes} \end{array}\right](\text{Mary}) \qquad ([\![smokes]\!], [\![Mary]\!])$$

Intransitive verbs as functions *and* sets?

Intransitive verbs are functions from individuals to truth values:

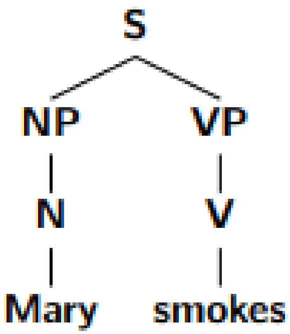
$[\![\textbf{dances}]\!] = f : D \rightarrow \{0, 1\}$

For all $x \in D$, $f(x) = 1$ iff x dances

$[\![\textbf{smokes}]\!] = f : D \rightarrow \{0, 1\}$

For all $x \in D$, $f(x) = 1$ iff x smokes

Recall: According to our intuition, there is another way to combine the two branches: **Set membership**.



S= 1 iff [[Mary]] $\in$

The individual Mary

$[\![\textbf{smokes}]\!] = \{x : x \in D \text{ and } x \text{ smokes } \}$

abbreviated as: $[\![\textbf{smokes}]\!] = \{x \in D : x \text{ smokes } \}$

Intransitive verbs can been seen as both functions and sets.

There must be some kind of relationship between sets and functions.

Sets and their characteristic functions

We use **characteristic function** to express the membership of the elements of any set S.

Mapping : Domain: The elements of D ⟶ Range: The set of truth-values {1,0}

(7) Assume domain D = {John Lennon, Paul McCartney, George Harrison, Ringo Starr, Jay-Z}

A:= {x: x is a member of the Beatles}

The characteristic function of A is

$$\begin{bmatrix} \text{John Lennon} & \longrightarrow & 1 \\ \text{Paul McCartney} & \longrightarrow & 1 \\ \text{George Harrison} & \longrightarrow & 1 \\ \text{Ringo Starr} & \longrightarrow & 1 \\ \text{Jay-Z} & \longrightarrow & 0 \end{bmatrix}$$

The 'table notation'

Relation between sets and their characteristic functions

Mapping of **characteristic functions**:

The elements of S ⟶ The set of truth-values {1,0}

This means:

Each subset S of the domain D defines such a function uniquely.

We write this as $\boldsymbol{char_{S,}}$ the characteristic function defined by S.

Any such function f (whose range is {0, 1}) corresponds to a unique subset S of the domain D.

We write this as $\boldsymbol{char_{f,}}$ the set S characterized by f.

$$\boldsymbol{char_S \longleftrightarrow char_f}$$

$char_S \longleftrightarrow char_f$: One-on-one correspondence

(8) Assume domain D = {John Lennon, Ringo Starr, Jay-Z, Chomsky, Spongebob}

There are some subsets of D:

A:= [[is a member of the Beatles]]={x: x is a member of the Beatles}

B: =[[raps]]={x: x raps}

C: =[[is a carton character]]={x: x is a carton character}

D: =[[is a linguisit]]={x: x is a linguisit}

What are the characteristic functions of the above subsets?

$char_S \longleftrightarrow char_f$: *One-on-one correspondence*

(8) Assume domain D = {John Lennon, Ringo Starr, Jay-Z, Chomsky, Spongebob}

There are some subsets of D:

A:= [[is a member of the Beatles]]={x: x is a member of the Beatles}

B: =[[is a rapper]]={x: x is a rapper}

$$char_A = \begin{bmatrix} \text{John Lennon} \longrightarrow 1 \\ \text{Paul McCartney} \longrightarrow 1 \\ \text{George Harrison} \longrightarrow 1 \\ \text{Ringo Starr} \longrightarrow 1 \\ \text{Jay-Z} \longrightarrow 0 \end{bmatrix} \qquad char_B = \begin{bmatrix} \text{John Lennon} \longrightarrow 0 \\ \text{Ringo Starr} \longrightarrow 0 \\ \text{Jay-Z} \longrightarrow 1 \\ \text{Chomsky} \longrightarrow 0 \\ \text{Spongebob} \longrightarrow 0 \end{bmatrix}$$

Exercise 3 : Characteristic function

(9)What is the characteristic function of set C and D? Use table notation.

C: =[[is a carton character]]={x: x is a carton character}

D: =[[is a linguist]]={x: x is a linguist}

Solutions: Exercise 3

C: =[[is a carton character]]={x: x is a carton character}

D: =[[is a linguist]]={x: x is a linguist}

$$char_C = \begin{bmatrix} \text{John Lennon} \rightarrow 0 \\ \text{Ringo Starr} \rightarrow 0 \\ \text{Jay-Z} \rightarrow 0 \\ \text{Chomsky} \rightarrow 0 \\ \text{Spongebob} \rightarrow 1 \end{bmatrix}$$

$$char_D = \begin{bmatrix} \text{John Lennon} \rightarrow 0 \\ \text{Ringo Starr} \rightarrow 0 \\ \text{Jay-Z} \rightarrow 0 \\ \text{Chomsky} \rightarrow 1 \\ \text{Spongebob} \rightarrow 0 \end{bmatrix}$$

$\mathit{char}_S \longleftrightarrow \mathit{char}_f$: *One-on-one correspondence*

(10) Assume domain D = {John Lennon, Ringo Starr, Jay-Z, Chomsky, Spongebob}

There are some functions whose range is {0, 1}:

[[is a member of the Beatles]] = $f_1 : D \longrightarrow \{0, 1\}$

For all $x \in D$, f(x) =1 iff x is a member of the Beatles.

[[raps]] = $f_2 : D \longrightarrow \{0, 1\}$

For all $x \in D$, f(x) =1 iff x raps.

char_{f1} = {x: x∈D and x is a member of the Beatles} = {John Lennon, Ringo Starr}

char_{f1} = {x: x∈D and x raps} = {Jay-Z}

Exercise 4: Characteristic function

(11) Assume domain D = {John Lennon, Ringo Starr, Jay-Z, Chomsky, Spongebob}

What are the sets characterized by f_3 and f_4?

[[lives in a pineapple under the sea]] = f_3: D $\longrightarrow$ {0, 1}

For all x ∈D, f(x)=1 iff x lives in a pineapple under the sea.

[[is a professional musician]] = f_4: D $\longrightarrow$ {0, 1}

For all x ∈D, f(x)=1 iff x a professional musician.

Solutions: Exercise 4

[[lives in a pineapple under the sea]]=$f_{3...}$

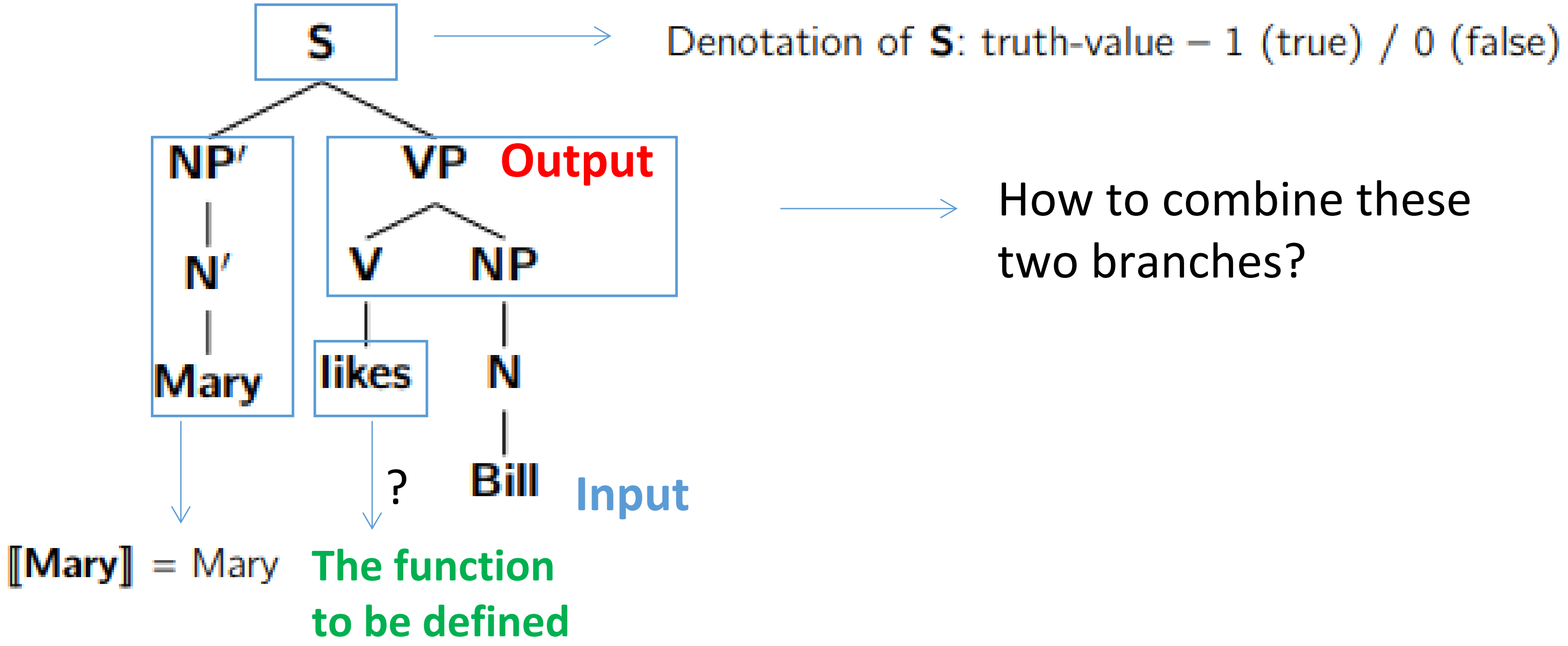
[[is a professional musician]]=$f_{4...}$

The sets characterized by f_3 and f_4 :

$char_{f3}$= {Spongebob}

$char_{f4}$= {John Lennon, Ringo Starr, Jay-Z}

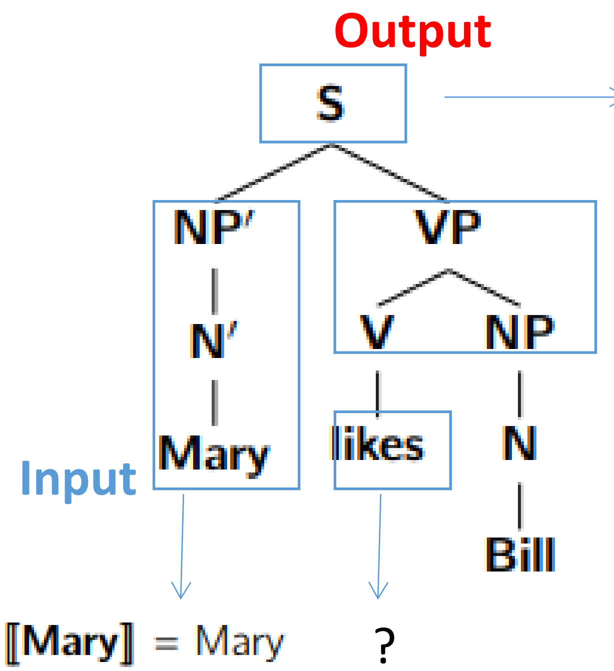
What about transitive verbs?



We need new rules to combine [[V]] and [[NP]] to the node [[VP]]. Bill likes? likes Bill?

Unlike intransitive verbs ‘smoke’, transitive verbs cannot denote a function from individuals to truth-values.

Branching VP-nodes



Denotation of **S**: truth-value – 1 (true) / 0 (false)

The function to be defined

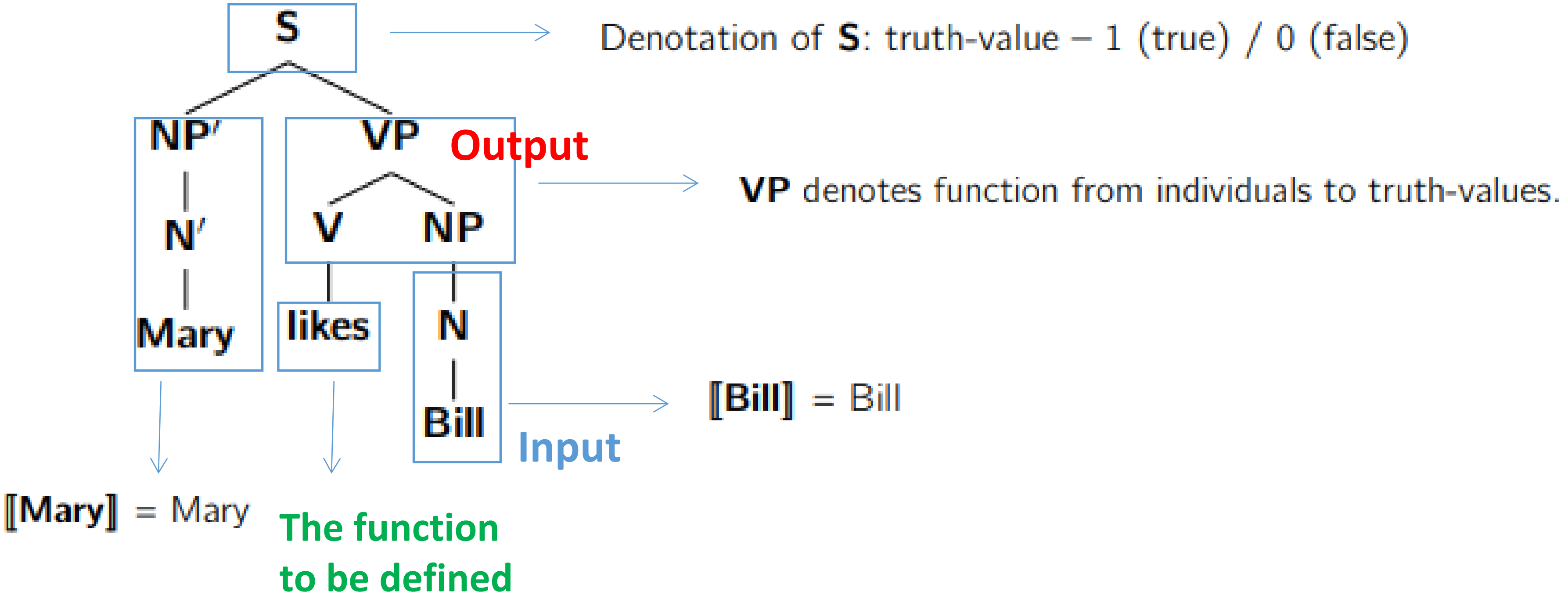
How to combine these two branches?

Same kind of function

char[[VP]]=char[[likes Bill]]= {x: x likes Bill} ⟷ *char[[smokes]]*= {x: x smokes}

VP denotes **a (characteristic) function from individuals to truth-values**. This is something we already know from intransitive verbs.

Filling in the blanks...



Transitive verbs 'likes' denote a function from individuals to functions from individuals to truth-values.

$$[\![\textbf{likes}]\!] = \begin{array}{l} f : D \to \{g : g \text{ is a function from } D \text{ to } \{1,0\}\} \\ \text{For all } x \in D,\ f(x) = \begin{array}{l} g_x : D \to \{1,0\} \\ \text{For all } y \in D,\ g_x(y) = 1 \text{ iff } y \text{ likes } x. \end{array} \end{array}$$

Function-valued functions as n-ary function.

(12) Assume D = {Fiona, Patsy, Jenny}

R_{like}= {< Fiona, Patsy >, < Patsy, Jenny >, < Jenny, Jenny >}

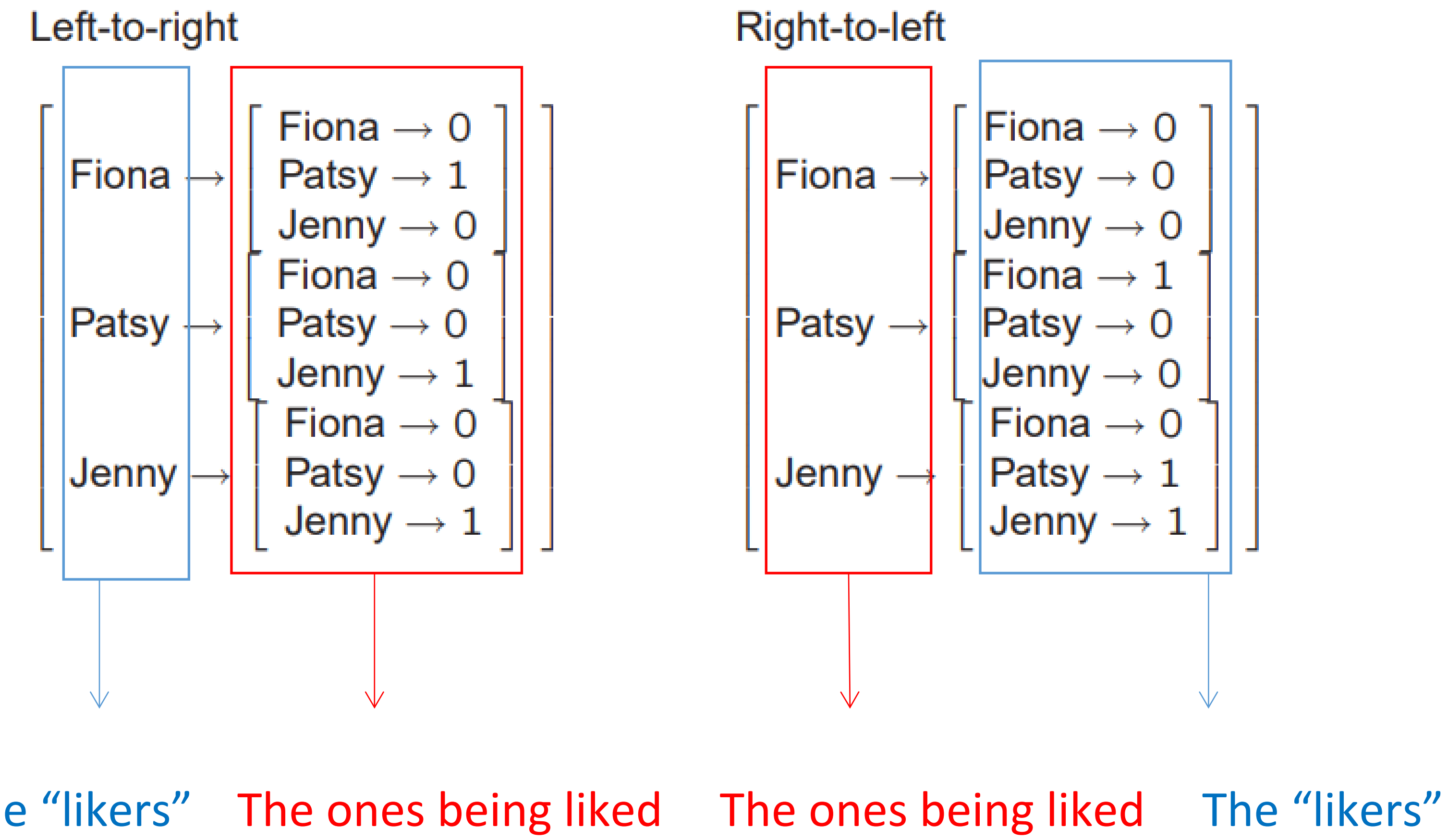
Characteristic function R_{like}=

$$\begin{bmatrix} < \text{Fiona, Fiona} > \rightarrow 0 \\ < \text{Fiona, Patsy} > \rightarrow 1 \\ < \text{Fiona, Jenny} > \rightarrow 0 \\ < \text{Patsy, Fiona} > \rightarrow 0 \\ < \text{Patsy, Patsy} > \rightarrow 0 \\ < \text{Patsy, Jenny} > \rightarrow 1 \\ < \text{Jenny, Fiona} > \rightarrow 0 \\ < \text{Jenny, Patsy} > \rightarrow 0 \\ < \text{Jenny, Jenny} > \rightarrow 1 \end{bmatrix}$$

This is a **binary function** that takes a pair of arguments. But our assumption before tells us that **“likes” denote a function takes exactly one individual as argument and maps it to a function**.

Reducing n-ary functions to unary ones

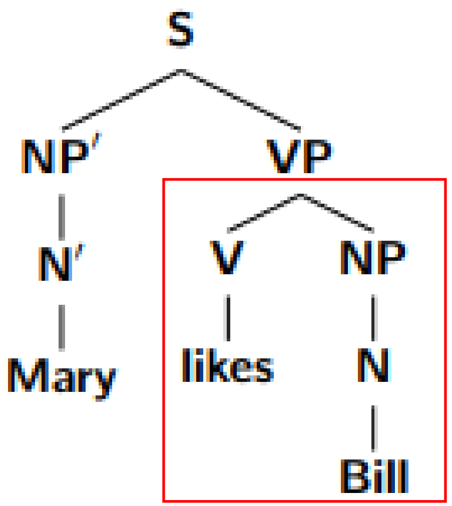
Schonfinkelization/currying: Turning n-ary functions into unary functions.



Schonfinkelization and grammar

From the ontological point of view, it does not matter which of the two ways we choose to model them.

But, does it make a difference fr of the grammar of English?



In hierarchical terms, the object "Bill" is the closest to the predicate "likes". Therefore, it should provide the argument for the function denoted by the predicate.

The **right-to-left Schonfinkelization** reflects the syntactic structure of English.

Exercise 5: Ditransitive verbs (assignment 3)

(13) Assume D = {Mary, John, Jane, Sue}

$R_{introduce}$= {<Mary, John, Sue>, <Mary, Jane, John>, <John, Jane, Sue>}

To be clear,<Mary, John, Sue> means ‘Mary introduces John to Sue’.

i. What is the characteristic function of $R_{introduce}$? Write this first as a ternary relation.

ii. There are two ways of using the **ditransitive** verb “introduce” in English:

a. Mary introduces John to Sue.

b. Mary introduces Sue John.

Which schönfinkelization of the characteristic function of $R_{introduce}$ does each of these two forms correspond to? Specify this as a unary function using the table notation

Exercise 5: Hint 1

i. We did some exercise to write the characteristic function of R_{like} as binary relation as shown in (14). What would the table notation look like for **ternary relation** with ordered triples like <Mary, John, Sue>?

(14)
$$\begin{bmatrix} <\text{Fiona, Fiona}> \rightarrow 0 \\ <\text{Fiona, Patsy}> \rightarrow 1 \\ <\text{Fiona, Jenny}> \rightarrow 0 \\ <\text{Patsy, Fiona}> \rightarrow 0 \\ <\text{Patsy, Patsy}> \rightarrow 0 \\ <\text{Patsy, Jenny}> \rightarrow 1 \\ <\text{Jenny, Fiona}> \rightarrow 0 \\ <\text{Jenny, Patsy}> \rightarrow 0 \\ <\text{Jenny, Jenny}> \rightarrow 1 \end{bmatrix}$$

Exercise 5: Hint 2

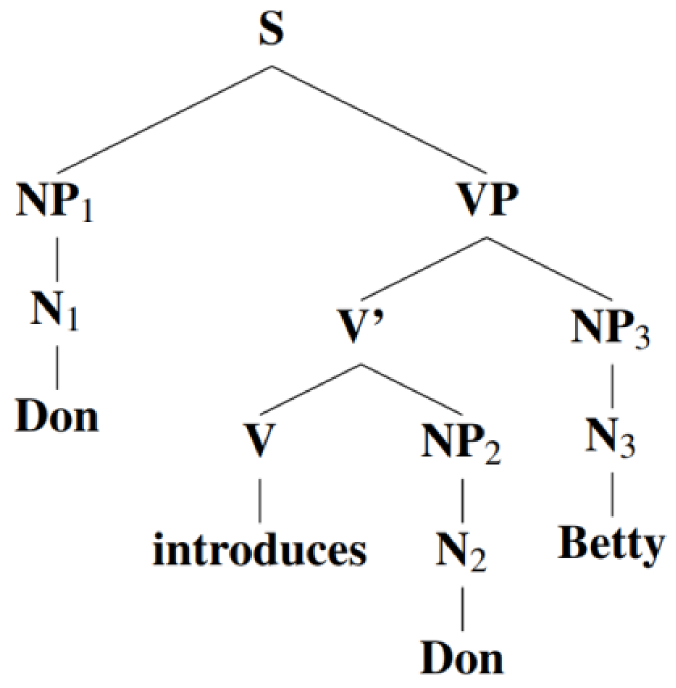
ii. There are two ways of using the ditransitive verb "introduce" in English:

a. Mary introduces John to Sue.

b. Mary introduces Sue John.

(1) a. **Don introduces himself to Betty.**

b.



Specify the schönfinkelization of both (iia) and (iib)

as a unary function as in (15).

Which kind of schönfinkelization does it match with?

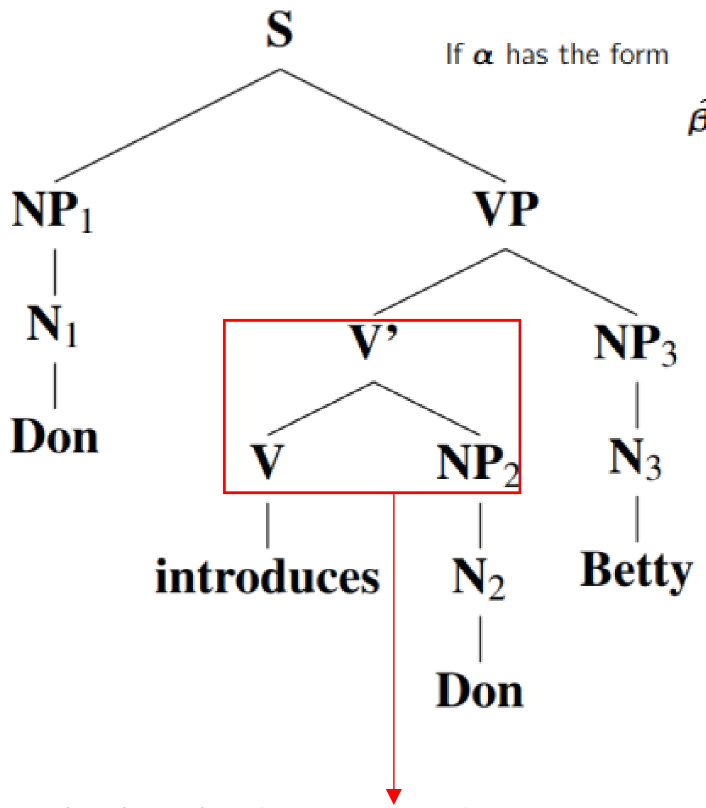
(15)

$$\begin{bmatrix} \text{Fiona} \rightarrow \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 1 \\ \text{Jenny} \rightarrow 0 \end{bmatrix} \\ \text{Patsy} \rightarrow \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 0 \\ \text{Jenny} \rightarrow 1 \end{bmatrix} \\ \text{Jenny} \rightarrow \begin{bmatrix} \text{Fiona} \rightarrow 0 \\ \text{Patsy} \rightarrow 0 \\ \text{Jenny} \rightarrow 1 \end{bmatrix} \end{bmatrix}$$

Exercise 5: Hint 3

(1) a. **Don introduces himself to Betty.**

b.



If α has the form $[_S\ \beta\ \gamma]$, then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

Which rule do we need to interpret V' i.e., to combine V and NP?

Semantic rules

S1 If $\boldsymbol{\alpha}$ has the form **S** , then $[\![\alpha]\!] = [\![\gamma]\!]([\![\beta]\!])$.

$\boldsymbol{\beta}$ $\boldsymbol{\gamma}$

S2 If $\boldsymbol{\alpha}$ has the form **NP**, then $[\![\alpha]\!] = [\![\beta]\!]$.

$\boldsymbol{\beta}$

S3 If $\boldsymbol{\alpha}$ has the form **VP**, then $[\![\alpha]\!] = [\![\beta]\!]$.

$\boldsymbol{\beta}$

S4 If $\boldsymbol{\alpha}$ has the form **N**, then $[\![\alpha]\!] = [\![\beta]\!]$.

$\boldsymbol{\beta}$

S5 If $\boldsymbol{\alpha}$ has the form **V**, then $[\![\alpha]\!] = [\![\beta]\!]$.

$\boldsymbol{\beta}$

S6 If $\boldsymbol{\alpha}$ has the form **S** , then $[\![\alpha]\!] = [\![\textbf{not}]\!]([\![\beta]\!])$

not $\boldsymbol{\beta}$

S7 If $\boldsymbol{\alpha}$ has the form **VP** , then $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

$\boldsymbol{\beta}$ $\boldsymbol{\gamma}$

Next time...

Please read through the lecture slides.

Negation, coordination, semantic types, λ-notation, conversion, type-driven interpretation

Thanks and see you next week!